A_4 flavour symmetry breaking scheme for understanding quark and neutrino mixing angles

Xiao-Gang He and Yong-Yeon Keum

NCTS/TPE, Department of Physics, National Taiwan University
Taipei, Taiwan, 10764, Republic of China
E-mail: `hexg@phys.ntu.edu.tw`, `yykeum@phys.ntu.edu.tw`

Raymond R. Volkas

School of Physics, Research Centre for High Energy Physics, The University of Melbourne
Victoria 3010, Australia
E-mail: `r.volkas@physics.unimelb.edu.au`

ABSTRACT: We propose a spontaneous A_4 flavour symmetry breaking scheme to understand the observed pattern of quark and neutrino mixing. The fermion mass eigenvalues are arbitrary, but the mixing angles are constrained in such a way that the overall patterns are explained while also leaving sufficient freedom to fit the detailed features of the observed values, including CP-violating phases. The scheme realises the proposal of Low and Volkas to generate zero quark mixing and tribimaximal neutrino mixing at tree level, with deviations from both arising from small corrections after spontaneous A_4 breaking. In the neutrino sector, the breaking is $A_4 \to Z_2$, while in the quark and charged-lepton sectors it is $A_4 \to Z_3 \cong C_3$. The full theory has A_4 completely broken, but the two different unbroken subgroups in the two sectors force the dominant mixing patterns to be as stated above. Radiative effects within each sector are shown to deviate neutrino mixing from tribimaximal, while maintaining zero quark mixing. Interactions between the two sectors — "cross-talk" — induce nonzero quark mixing, and additional deviation from tribimaximal neutrino mixing. We discuss the vacuum alignment challenge the scenario faces, and suggest three generic ways to approach the problem. We follow up one of those ways by sketching how an explicit model realising the symmetry breaking structure may be constructed.

Contents

1. Introduction

The explanation of flavour is one of the most profound goals in the construction of standard model (SM) extensions. There are several aspects to the overall puzzle: Why three families of quarks and leptons? Why the specific mixing patterns observed? Can we understand quark and lepton mass eigenvalues? Why are neutrinos so light? A priori, it is not clear if these aspects of the flavour problem should be treated organically, or they can be solved piecemeal. In this paper, we show that the observed mixing angle patterns suggest an underlying flavour symmetry breaking structure based on the discrete group A_4, while also incorporating the see-saw explanation for why neutrinos are especially light.[1]

It is interesting that these features can be understood without needing to know why the fermions come in three families, and without drawing any connection with an explanation for the mass eigenvalues. The latter will be arbitrary in our scheme. Note that in the neutrino sector, most of our current knowledge about masses and mixing angles comes from neutrino oscillation data which provide no direct information about the absolute values of neutrino masses, while detailed information about neutrino mixing has been obtained. The analysis to be presented may provide crucial information for understanding

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[1]A_4 is the alternating group of order four, defined as the set of all even permutations of four objects. Geometrically, it is the symmetry group of the tetrahedron. See the appendix A for a list of the basic results we will use in this paper, and refs. $[1-5]$ for further discussion.

the mixing mechanism in both the lepton and quark sectors. The present neutrino data can be accommodated fairly well [6] by the so-called tribimaximal mixing [7] matrix. We shall therefore use tribimaximal neutrino mixing as the lowest order approximation and study allowed deviations within our A_4 structure. In the quark sector, we set up the mixing matrix to be the identity matrix at lowest order, and then generate non-trivial mixing through corrections.

The main point of this paper is to argue for a certain symmetry-breaking structure, rather than to advocate a specific model realising it (we shall call this the "dynamical completion" problem). Given our present ignorance, at the experimentally-verified level, of the precise dynamics nature chose to break the electroweak symmetry, we feel that it is of considerable value to begin by studying flavour symmetry breaking in a way as independent of dynamics as possible. The resulting symmetry-based understanding may have more long-term value than, for example, explicit Higgs-potential realisations. Indeed, the insights so gained can be used to guide subsequent model-building. The latter may encompass conventional Higgs models, as well as brane-world realisations and other schemes, according to the skill and taste of the model-builder.

Nevertheless, we shall find it convenient to think in terms of Higgs fields and their expectation values, and for the sake of completeness we shall also briefly discuss the dynamical completion issue and suggest possible solutions.

Our ideas were seeded by two considerations from the literature. First, several authors have recently explored very interesting connections between flavour A_4 [2−5] and the tribimaximal neutrino mixing matrix [7]. Second, a conjecture has been proposed by Low and Volkas on the relationship between quark and lepton mixing [8]. It is often remarked that these two sectors are jarringly different: quark mixing reveals three small mixing angles, while lepton mixing requires two large angles (one consistent with maximal), and one small angle (consistent with zero). The conjecture begins by noting that each of the mixing matrices is of the form $V_1^\dagger V_2$, where the V_i are the left-diagonalisation matrices of the species involved. The proposal is that due to symmetry, the V_i matrices for up-quarks and down-quarks are identical, leading to a trivial Cabibbo-Kobayashi-Maskawa (CKM) matrix [9], but with each quark V_i having large off-diagonal entries. Quark mixing is "trying" to be large, but the effects are exactly cancelled due to a symmetry. In the lepton sector, one then proposes that one of the diagonalisation matrices – the neutrino one in the original conjecture and subsequently in the present paper – takes a different form from the other three due to different symmetry constraints. There is no perfect cancellation, and thus Maki-Nakagawa-Sakata-Pontecorvo (MNSP) [10] mixing contains large angles. To agree with experiment, and to hold the promise of an underlying symmetry, the tribimaximal form for the MNSP matrix is selected. Deviations from diagonal CKM and tribimaximal MNSP then arise from corrections after symmetry breaking. We show how this conjecture can be realised in our A_4 scheme.

Our approach delivers some interesting quantitative relations between mixing angles and CP violating phases, and relates the neutrino mixing angle θ_{13} to other deviations from the tribimaximal MNSP form.

In the next section 2 we define the field content of the scheme, and explain how the

dominant tree-level mixing matrices arise. Section 3 then explains how a class of radiative corrections — those intrinsic to the neutrino sector on its own, and the charged-fermion sector on its own — alters the tree-level picture: the tribimaximal pattern is modified, but CKM mixing is still absent. In section 4, interactions between the sectors are used to generate CKM mixing and, generically, to induce additional deviation for tribimaximal neutrino mixing. Section 5 discusses the dynamical completion challenge, while section 6 is a conclusion. Appendix A states the basic A_4 results we shall use, appendix B lists the Higgs potential of the minimal model, while appendix C discusses a supersymmetric dynamical completion.

2. The scheme and tree-level results

The symmetry group of our scheme is $G \otimes X$, where

$$G = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes A_4, \tag{2.1}$$

and the usual SM gauge group is augmented by an A_4 flavour symmetry plus an auxilliary symmetry X whose nature and role shall be discussed fully below. The three families of quarks and leptons are placed in the following representations of G:

$$Q_L \sim \left(3, 2, \frac{1}{3}\right) (\underline{\mathbf{3}}) \qquad \ell_L \sim (1, 2, -1) (\underline{\mathbf{3}})$$

$$u_R \oplus u'_R \oplus u''_R \sim \left(3, 1, \frac{4}{3}\right) (\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'') \qquad \nu_R \sim (1, 1, 0) (\underline{\mathbf{3}}) \tag{2.2}$$

$$d_R \oplus d'_R \oplus d''_R \sim \left(3, 1, -\frac{2}{3}\right) (\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'') \qquad e_R \oplus e'_R \oplus e''_R \sim (1, 1, -2) (\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'')$$

where the A_4 notation is explained in the appendix A, and the G_{SM} notation is standard. Models with similar A_4 assignments for the leptons and Higgs fields have been considered with a different emphasis in ref.[5]. Notice that the right-handed neutrinos are assigned to a $\underline{\mathbf{3}}$, whereas the right-handed charged-fermions are each given a $\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}''$ structure. The Higgs field assignments are

$$\Phi \sim (1, 2, -1) (\underline{\mathbf{3}}), \quad \phi \sim (1, 2, -1) (\underline{\mathbf{1}}), \quad \chi \sim (1, 1, 0) (\underline{\mathbf{3}}). \tag{2.3}$$

The $G \otimes X$ invariant Yukawa Lagrangian is

$$\begin{aligned}
\mathcal{L}_{\text{Yuk}} = {} & \lambda_u \, (\overline{Q}_L \Phi)_{\underline{\mathbf{1}}} \, u_R + \lambda'_u (\overline{Q}_L \Phi)_{\underline{\mathbf{1}}'} \, u''_R + \lambda''_u (\overline{Q}_L \Phi)_{\underline{\mathbf{1}}''} \, u'_R + \\
& + \lambda_d (\overline{Q}_L \tilde{\Phi})_{\underline{\mathbf{1}}} \, d_R + \lambda'_d (\overline{Q}_L \tilde{\Phi})_{\underline{\mathbf{1}}'} \, d''_R + \lambda''_d (\overline{Q}_L \tilde{\Phi})_{\underline{\mathbf{1}}''} \, d'_R + \\
& + \lambda_\nu (\overline{\ell}_L \nu_R)_{\underline{\mathbf{1}}} \, \phi + M [\overline{\nu}_R (\nu_R)^c]_{\underline{\mathbf{1}}} + \lambda_\chi [\overline{\nu}_R (\nu_R)^c]_{\underline{\mathbf{3}}_s} \cdot \chi + \\
& + \lambda_e (\overline{\ell}_L \tilde{\Phi})_{\underline{\mathbf{1}}} \, e_R + \lambda'_e (\overline{\ell}_L \tilde{\Phi})_{\underline{\mathbf{1}}'} \, e''_R + \lambda''_e (\overline{\ell}_L \tilde{\Phi})_{\underline{\mathbf{1}}''} \, e'_R + h.c.
\end{aligned} \tag{2.4}$$

where $\tilde{\Phi} \equiv i\tau_2 \Phi^*$. This rather busy-looking equation actually has a quite simple structure. Each charged fermion sector has three independent Yukawa terms, all involving the A_4 triplet Higgs field Φ but not the flavour-singlet ϕ. By construction, the neutrino sector is

different. The neutrino Dirac term is governed by a single coupling constant and involves ϕ, while the right-handed Majorana sector contains one bare Majorana mass M and a single Yukawa coupling term to the Higgs electroweak-singlet χ (which is an A_4 triplet).[2] All told, there are only twelve (a priori complex) parameters to describe the masses and mixings of nine Dirac and six Majorana fermions. The restrictions will prove to be rather interesting.

The Yukawa Lagrangian of eq. (2.4) has the additional symmetry $U(1)_X$, where ℓ_L, e_R, e'_R, e''_R and ϕ carry $X = 1$, while all other fields have $X = 0$. This non-flavour symmetry ensures that the $G_{SM} \otimes A_4$ invariant Yukawa term $\overline{\ell}_L \nu_R \Phi$ is absent from the Lagrangian. Since $U(1)_X$ is anomalous, it cannot be gauged. The Goldstone boson arising from spontaneous $U(1)_X$ breaking through $\langle \phi \rangle \neq 0$ is phenomenologically disallowed, so we will ultimately have to break $U(1)_X$ explicitly down to a discrete subgroup that is sufficient to prevent the unwanted Yukawa term (see later).

Writing out the charged-fermion $f = u, d, e$ Yukawa invariants explicitly using the rules A.5-A.7 in the appendix, one finds that each of the three mass matrix terms has the form

$$\left(\overline{f}_{1L}, \overline{f}_{2L}, \overline{f}_{3L} \right) \begin{pmatrix} \lambda v_1 & \lambda' v_1 & \lambda'' v_1 \\ \lambda v_2 & \omega \lambda' v_2 & \omega^2 \lambda'' v_2 \\ \lambda v_3 & \omega^2 \lambda' v_3 & \omega \lambda'' v_3 \end{pmatrix} \begin{pmatrix} f_R \\ f''_R \\ f'_R \end{pmatrix} + h.c. \tag{2.5}$$

where $\langle \Phi^0 \rangle = (v_1, v_2, v_3)$ is the vacuum expectation value (VEV) pattern for Φ, the v_i are taken to be relatively real, and the λ's have a suppressed subscript f. The numerical subscripts $1, 2, 3$ denote A_4 components, as in the appendix.

For the special VEV pattern

$$v_1 = v_2 = v_3 \equiv v \tag{2.6}$$

each of these mass matrices M_f factorises as per

$$M_f = U(\omega) \begin{pmatrix} \sqrt{3}\lambda_f v & 0 & 0 \\ 0 & \sqrt{3}\lambda'_f v & 0 \\ 0 & 0 & \sqrt{3}\lambda''_f v \end{pmatrix}, \tag{2.7}$$

so that the left-diagonalisation matrices $V_L^{u,d,e}$ for, respectively, the upquark, down-quark and charged-lepton sectors are identical and equal to the unitary "trimaximal mixing matrix"

$$U(\omega) = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}. \tag{2.8}$$

Notice that all nine mass eigenvalues are a priori arbitrary, despite the totally prescribed diagonalisation matrices. This is an example of "form diagonalisability", a term coined in ref.[8] to describe exactly this situation. The process here is a complete contrast to the popular strategy of relating mixing angles to mass ratios.

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[2]Note that the $\underline{\mathbf{3}}a$ product of $\overline{\nu}_R$ and $(\nu_R)^c$ identically vanishes.

One immediately finds that, at this order, the chosen A_4 structure of the field content and the $\langle \Phi \rangle$ vacuum forces the CKM matrix to be the identity:

$$V_{CKM} = V_L^{d\dagger} V_L^u = U(\omega)^\dagger U(\omega) = 1. \tag{2.9}$$

The vacuum is a very special one, as it induces the breakdown

$$A_4 \to Z_3 \cong C_3 = \{1, c, a\}, \tag{2.10}$$

where $\cong$ denotes "isomorphism". The flavour group is not broken completely at this stage, but only to the three-fold subgroup that cyclically permutes the three A_4 triplet basis states without changing their signs (see eq. (A.1)). The $\underline{1}'$ and $\underline{1}''$ spaces transform under this subgroup exactly as they do under the full group A_4. As we show below, the C_3 remnant, if forever unbroken, is powerful enough to ensure that the CKM matrix remains trivial to all orders.

Now to one of our main points: It is quite possible that the reason why the observed CKM matrix is nearly the identity is the hierarchical breaking

$$A_4 \to C_3 \to \text{nothing}, \tag{2.11}$$

with the small mixing angles generated by higher-order effects after the relatively weak subsequent breaking of the residual C_3. Before taking this line of thought further, we need to examine the neutrino sector.

The neutrino Dirac mass matrix is different from that of the charged-leptons, being derived from the Yukawa term $\overline{\ell}_L \nu_R \phi$, where the fermion bilinear sees the two A_4 triplets coupling to the singlet. From eq. (A.5), one simply gets that the Dirac mass matrix is proportional to the 3×3 identity matrix,

$$M_\nu^D = \lambda_\nu \, v_\phi \, 1 \equiv m_\nu^D \, 1, \tag{2.12}$$

where $\langle \phi^0 \rangle = v_\phi$. The right-handed neutrino bare Majorana mass term is similarly trivial, being M times the identity. The required non-trivial structure is supplied by the Yukawa coupling to χ, which expanded out is

$$\lambda_\chi \, \left(\overline{\nu}_{1R}, \overline{\nu}_{2R}, \overline{\nu}_{3R} \right) \begin{pmatrix} 0 & \chi_3 & \chi_2 \\ \chi_3 & 0 & \chi_1 \\ \chi_2 & \chi_1 & 0 \end{pmatrix} \begin{pmatrix} (\nu_{1R})^c \\ (\nu_{2R})^c \\ (\nu_{3R})^c \end{pmatrix}. \tag{2.13}$$

We now make our second key assumption about A_4 breaking: we want

$$\langle \chi_1 \rangle = \langle \chi_3 \rangle = 0, \quad \langle \chi_2 \rangle \equiv v_\chi \neq 0, \tag{2.14}$$

so that the full 6×6 neutrino mass matrix is

$$\begin{pmatrix} 0 & 0 & 0 & m_\nu^D & 0 & 0 \\ 0 & 0 & 0 & 0 & m_\nu^D & 0 \\ 0 & 0 & 0 & 0 & 0 & m_\nu^D \\ m_\nu^D & 0 & 0 & M & 0 & M_\chi \\ 0 & m_\nu^D & 0 & 0 & M & 0 \\ 0 & 0 & m_\nu^D & M_\chi & 0 & M \end{pmatrix}, \tag{2.15}$$

where $M_\chi \equiv \lambda_\chi v_\chi$. Note that M and M_χ are in general complex numbers with a relative phase difference. In the see-saw limit $|M|, |M_\chi| \gg m_\nu^D$, the effective 3×3 mass matrix M_L for the light neutrino sector is simply

$$M_L = -M_\nu^D M_R^{-1} (M_\nu^D)^T = -\frac{(m_\nu^D)^2}{M} \begin{pmatrix} \frac{M^2}{M^2 - M_\chi^2} & 0 & -\frac{M M_\chi}{M^2 - M_\chi^2} \\ 0 & 1 & 0 \\ -\frac{M M_\chi}{M^2 - M_\chi^2} & 0 & \frac{M^2}{M^2 - M_\chi^2} \end{pmatrix}, \qquad (2.16)$$

whose diagonalisation matrix is simply

$$V_L^\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix}. \qquad (2.17)$$

The MNSP matrix, at this order, is then

$$V_{MNSP} = V_L^{e\dagger} V_L^\nu = U(\omega)^\dagger V_L^\nu = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{\omega}{\sqrt{6}} & \frac{\omega}{\sqrt{3}} & -\frac{e^{i\pi/6}}{\sqrt{2}} \\ -\frac{\omega^2}{\sqrt{6}} & \frac{\omega^2}{\sqrt{3}} & \frac{e^{-i\pi/6}}{\sqrt{2}} \end{pmatrix} \qquad (2.18)$$

which, up to phases, is tribimaximal and hence fits the neutrino oscillation data well. These neutrino results are the same as in the recently explored scenario of ref. [3]; we refer the reader to this paper for a more extended phenomenological discussion.

In the neutrino sector, the flavour breaking pattern driven by $\langle \chi \rangle$ is

$$A_4 \to Z_2 = \{1, r_2\}. \qquad (2.19)$$

This Z_2 subgroup does not commute with the C_3 subgroup of the charged-fermion sector. The neutrino and charged-fermion sectors form "parallel worlds" of flavour symmetry breaking. These parallel symmetry breaking worlds cannot be sequestered from each other completely, of course. For the theory as a whole, A_4 is completely broken.

After flavour symmetry breaking, higher-order and radiative effects will in general create terms that violate A_4. We will divide these higher-order effects into two classes: those that involve effects *within each sector*, and those that involve *interactions between the two sectors*. The former are precisely the effects that preserve C_3 and Z_2, respectively, for the charged-lepton and neutrino sectors. The latter violate A_4 completely. We wish to see how these different classes correct the CKM amd MNSP matrices from the trivial and tribimaximal forms, respectively. We shall work in as dynamics-independent a way as possible.

3. Corrections within each sector after flavour symmetry breaking.

Let us start with the charged-fermion sector. At the bare Lagrangian level, the only Yukawa terms allowed are those invariant under A_4. After A_4 spontaneously breaks to C_3 in this sector, higher-order effects will generate Yukawa terms that violate A_4 but respect C_3. We now write down all those terms.

Under $C_3 = \{1, c, a\}$, the triplets Q_L, ℓ_L and Φ transform as per

$$c : (1, 2, 3) \rightarrow (3, 1, 2) \quad \text{and} \quad a : (1, 2, 3) \rightarrow (2, 3, 1), \tag{3.1}$$

where $1, 2, 3$ denote the triplet entries, as before. The A_4 singlets f_R become C_3 singlets ($f = u, d, e$ as before), while the non-trivial one-dimensional A_4-plets f'_R and f''_R transform thus:

$$f'_R \begin{cases} \xrightarrow{c} \omega\, f'_R \\ \xrightarrow{a} \omega^2 f'_R \end{cases} \quad \text{and} \quad f''_R \begin{cases} \xrightarrow{c} \omega^2 f''_R \\ \xrightarrow{a} \omega\, f''_R \end{cases} \tag{3.2}$$

The previously allowed A_4 invariant $(\overline{f}_{1L}\Phi_1^0 + \overline{f}_{2L}\Phi_2^0 + \overline{f}_{3L}\Phi_3^0)\, f_R$ (where the Φ_i^0 generically denote the charge-neutral fields within Φ and $\tilde{\Phi}$) is now supplemented with the following terms that violate A_4 but respect C_3:

$$\begin{aligned} (\overline{f}_{1L}\Phi_2^0 + \overline{f}_{2L}\Phi_3^0 + \overline{f}_{3L}\Phi_1^0)\, f_R \\ (\overline{f}_{1L}\Phi_3^0 + \overline{f}_{2L}\Phi_1^0 + \overline{f}_{3L}\Phi_2^0)\, f_R. \end{aligned} \tag{3.3}$$

Similarly, the A_4 invariants $(\overline{f}_{1L}\Phi_1^0 + \omega\overline{f}_{2L}\Phi_2^0 + \omega^2\overline{f}_{3L}\Phi_3^0)\, f''_R$ and $(\overline{f}_{1L}\Phi_1^0 + \omega^2\overline{f}_{2L}\Phi_2^0 + \omega\overline{f}_{3L}\Phi_3^0)\, f'_R$ are joined by

$$\begin{aligned} (\overline{f}_{1L}\Phi_2^0 + \omega\overline{f}_{2L}\Phi_3^0 + \omega^2\overline{f}_{3L}\Phi_1^0)\, f''_R \\ (\overline{f}_{1L}\Phi_3^0 + \omega\overline{f}_{2L}\Phi_1^0 + \omega^2\overline{f}_{3L}\Phi_2^0)\, f''_R \\ (\overline{f}_{1L}\Phi_2^0 + \omega^2\overline{f}_{2L}\Phi_3^0 + \omega\overline{f}_{3L}\Phi_1^0)\, f'_R \\ (\overline{f}_{1L}\Phi_3^0 + \omega^2\overline{f}_{2L}\Phi_1^0 + \omega\overline{f}_{3L}\Phi_2^0)\, f'_R. \end{aligned} \tag{3.4}$$

Each of the new terms comes, generically, with a different coupling constant. It is interesting, though, that despite all these new Yukawa terms, the mass matrices retain the form of eq. (2.7) once the C_3-preserving VEV pattern $\langle\Phi_1^0\rangle = \langle\Phi_2^0\rangle = \langle\Phi_3^0\rangle \equiv v$ is used. This means that the left-diagonalisation matrices for the u, d and e sectors remain trimaximal, and hence the CKM matrix remains trivial. We have thus demonstrated that it is the C_3 subgroup of A_4 that is responsible for preventing quark mixing. The origin of CKM mixing must then arise from C_3 breaking, which in the spirit of economy one may wish to extract from the neutrino sector (though this is not mandatory – one may also extend the theory).

We now turn to the neutrino sector. It is easy to see that the minus sign associated with the unbroken Z_2 transformations keeps the $(1, 2)$, $(2, 1)$, $(2, 3)$ and $(3, 2)$ entries of both the neutrino Dirac and right-handed Majorana mass matrices zero. However, these two matrices need not be proportional to the identity any longer, as that feature was driven by the now broken A_4. We now have, in general, that

$$M_\nu^D = \lambda_\nu \langle\phi\rangle \left.\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right|_{\text{bare}} + \left.\begin{pmatrix} \epsilon_{11} & 0 & \epsilon_{13} \\ 0 & \epsilon_{22} & 0 \\ \epsilon_{31} & 0 & \epsilon_{33} \end{pmatrix}\right|_{\text{h.o.}} \tag{3.5}$$

and the bare right-handed Majorana mass matrix is

$$M \left.\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right|_{\text{bare}} + \left.\begin{pmatrix} \epsilon'_{11} & 0 & \epsilon'_{13} \\ 0 & \epsilon'_{22} & 0 \\ \epsilon'_{31} & 0 & \epsilon'_{33} \end{pmatrix}\right|_{\text{h.o.}} \tag{3.6}$$

where subscript "h.o." stands for "higher order", with the ϵ and ϵ' entries being small. The $\nu_R - \chi$ coupling terms now also contain the independent Z_2 invariants

$$\overline{\nu}_{2R}(\nu_{3R})^c\chi_{1,3}, \quad \overline{\nu}_{3R}(\nu_{1R})^c\chi_2, \quad \overline{\nu}_{1R}(\nu_{2R})^c\chi_{3,1},$$
$$\overline{\nu}_{1R}(\nu_{1R})^c\chi_2, \quad \overline{\nu}_{2R}(\nu_{2R})^c\chi_2, \quad \overline{\nu}_{3R}(\nu_{3R})^c\chi_2. \tag{3.7}$$

Inputting the Z_2-preserving vacuum of eq (2.14), we see that the new terms involve corrections to the (i,i) and $(1,3) = (3,1)$ entries in the right-handed Majorana mass matrix. In total then, we have that the right-handed Majorana mass terms are

$$\left.\begin{pmatrix} M & 0 & M_\chi \\ 0 & M & 0 \\ M_\chi & 0 & M \end{pmatrix}\right|_{\text{bare}} + \left.\begin{pmatrix} \epsilon''_{11} & 0 & \epsilon''_{13} \\ 0 & \epsilon''_{22} & 0 \\ \epsilon''_{31} & 0 & \epsilon''_{33} \end{pmatrix}\right|_{\text{h.o.}} \tag{3.8}$$

It is obvious then that the effective M_L is additively corrected from eq. (2.16) by

$$M_L \rightarrow M_L + \left.\begin{pmatrix} \delta_{11} & 0 & \delta_{13} \\ 0 & \delta_{22} & 0 \\ \delta_{13} & 0 & \delta_{33} \end{pmatrix}\right|_{\text{h.o.}}, \tag{3.9}$$

where, in general, the δ_{ij} are complex. The neutrino left-diagonalisation matrix is now corrected to

$$V_L^\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & e^{i\alpha_3} \end{pmatrix}, \tag{3.10}$$

where

$$\theta = \frac{\pi}{4} + \delta \tag{3.11}$$

and $\delta \ll 1$. The phases α_i can be absorbed into the neutrino mass eigenstate fields, but the phase β, given by

$$\beta = \text{Arg}(M + \delta_{33}) - \text{Arg}(M + \delta_{11}), \tag{3.12}$$

is important because it will contribute to CP violation in neutrino oscillations.

The MNSP matrix becomes

$$V_{MNSP} = U(\omega)^\dagger V_L^\nu = \frac{1}{\sqrt{3}} \begin{pmatrix} c + se^{i\beta} & 1 & ce^{i\beta} - s \\ c + \omega se^{i\beta} & \omega^2 & \omega ce^{i\beta} - s \\ c + \omega^2 se^{i\beta} & \omega & \omega^2 ce^{i\beta} - s \end{pmatrix}, \tag{3.13}$$

where $c \equiv \cos\theta$ and $s \equiv \sin\theta$. The middle column is uncorrected at this level, a nonzero U_{e3} element is generated, and there are other small deviations from exact tribimaximal mixing.

CP violation in neutrino oscillations is generated at this level. The Jarlskog invariant is

$$\text{Im}[V_{11}\, V_{12}^*\, V_{21}^*\, V_{22}] = \frac{1}{9}(\cos 2\theta - \sin 2\theta \sin\beta)\sin\left(\frac{2\pi}{3}\right) \tag{3.14}$$

where the V_{ij} denote the entries of V_{MNSP}.

4. Interactions between the sectors after flavour symmetry breaking

So far, we have neglected interactions between the charged-fermion (plus Φ) sector and the neutrino (plus χ and ϕ) sector, the two parallel worlds of flavour symmetry breaking. The neutrino-world A_4 breaking is sufficient to generate realistic neutrino mass and mixing phenomenology while at the same time explaining why the dominant mixing pattern is tribimaximal. The charged-fermion world, however, has an interesting residual C_3 symmetry that prevents the generation of quark mixing. This is pleasing at lowest order because of the known fact that CKM angles are small; clearly, however, the C_3 subgroup must be slightly broken to achieve a fully realistic quark sector.

As remarked earlier, the full theory does in fact have broken C_3, as the neutrino sector does not respect it. This suggests that one should look to C_3 breaking mediated to the quark sector from the neutrino/χ sector as the natural source for quark mixing, which we may term "cross-talk".

The details of the cross-talk depend on the specific dynamics. Since we want to use symmetry on its own as much as possible, we adopt an effective operator approach.[3] This allows us to identify what the symmetry breakdown structure in principle allows by the way of dynamical outcomes. If realistic quark mixing were to be allowed, then that would motivate the construction of explicit models.

The effective operators relevant for quark mixing are at dimension-five. Schematically, they are of the forms

$$\overline{Q}_L \, u_R \, \Phi \, \chi, \quad \overline{Q}_L \, u'_R \, \Phi \, \chi, \quad \overline{Q}_L \, u''_R \, \Phi \, \chi$$
$$\overline{Q}_L \, d_R \, \tilde{\Phi} \, \chi, \quad \overline{Q}_L \, d'_R \, \tilde{\Phi} \, \chi, \quad \overline{Q}_L \, d''_R \, \tilde{\Phi} \, \chi, \tag{4.1}$$

plus similar operators with $\Phi \to \phi$. The VEV of χ communicates C_3 breaking to the quarks through these operators. The operators involving ϕ are eliminated by the $U(1)_X$ symmetry (or the relevant discrete subgroup thereof), so we need only consider the set in eq. (4.1). Each operator is suppressed either by a high mass scale M_{inter} that characterises the dynamical interactions between the two sectors (or a set of such scales), or by small coupling constants controlling those interactions.

Looked at more carefully, we see that each of the above yields two independent A_4 invariants. For example, $\overline{Q}_L \, u_R \, \Phi \, \chi$ schematically denotes the independent terms

$$[(\overline{Q}_L \, \Phi)_{\underline{3}_s} \, \chi]_{\underline{1}} \, u_R \quad \text{and} \quad [(\overline{Q}_L \, \Phi)_{\underline{3}_a} \, \chi]_{\underline{1}} \, u_R. \tag{4.2}$$

Expanding out these terms, and inserting the VEVs of eqs. (2.6) and (2.14), it is easy to see that the dimension-five operators yield corrections to the quark mass matrices of the forms

$$\Delta M_{u,d} = \begin{pmatrix} x_1^{u,d} & x_2^{u,d} & x_3^{u,d} \\ 0 & 0 & 0 \\ y_1^{u,d} & y_2^{u,d} & y_3^{u,d} \end{pmatrix} \tag{4.3}$$

---

[3]Note that the analysis of the preceding section could also have been phrased in the language of effective operators. For instance, the $\nu_R - \chi$ terms that violate A_4 but preserve Z_2 can originate from higher-dimensional operators such as $\overline{\nu}_R (\nu_R)^c \chi^{n+1}$ where n of the χ's are replaced by their VEVs.

where the entries are in general complex. The corrected mass matrices are then easily seen to be

$$
\begin{aligned}
M + \Delta M &= U(\omega)\sqrt{3} \begin{pmatrix} \lambda v + (x_1 + y_1)/3 & (x_2 + y_2)/3 & (x_3 + y_3)/3 \\ (x_1 + \omega y_1)/3 & \lambda' v + (x_2 + \omega y_2)/3 & (x_3 + \omega y_3)/3 \\ (x_1 + \omega^2 y_1)/3 & (x_2 + \omega^2 y_2)/3 & \lambda'' v + (x_3 + \omega^2 y_3)/3 \end{pmatrix} \\
&\equiv U(\omega)V_L \begin{pmatrix} m_{u,d} & 0 & 0 \\ 0 & m_{c,s} & 0 \\ 0 & 0 & m_{t,b} \end{pmatrix} V_R^\dagger
\end{aligned} \tag{4.4}
$$

where some of the u, d labels have, for clarity, been suppressed. The left-diagonalisation matrices are now $U(\omega)V_L^{u,d}$ instead of just $U(\omega)$, where $V_L^{u,d}$ are nearly diagonal if the x's and y's are smaller in magnitude than the λv's. This means that the CKM matrix is now given by

$$
V_{CKM} = [U(\omega)V_L^d]^\dagger [U(\omega)V_L^u] = V_L^{d\dagger} V_L^u \neq 1. \tag{4.5}
$$

In general, there is enough freedom in $V_L^{u,d}$ to fit the observed CKM matrix, while also, of course, explaining why it is nearly the identity.

So, we conclude that the overall structure allows the χ-sector to seed the C_3 breaking required to generate quark mixing. To understand more detailed features of the CKM matrix, such as the relative magnitudes of the $(1,2)$, $(2,3)$ and $(1,3)$ entries, it appears one would need an explicit fundamental theory with the correct relative sizes for the x's and y's.

Generically, one also expects the charged-fermion-Φ sector to feed through into the neutrino sector and hence to provide additional deviation from tribimaximal mixing. The extent to which this would happen is model-dependent. It would be pleasing to construct a model where these effects were actually very small, in order to preserve the appealing neutrino mixing pattern described in the previous section.

5. Ideas for an underlying dynamics

We have described an approach to understanding flavour mixing angles driven as much as possible purely by spontaneous A_4 symmetry breaking and its generic consequences. We made limited use of the language of Higgs fields and their expectation values for concreteness and convenience, but otherwise tried to be dynamically non-committed.

Eventually, though, one wants a dynamical completion for our scenario. The possibilities for this may only be limited by the creative powers of the model builder. However, there is an important challenge: it is not trivial to ensure that the different vacuum alignments of $\langle \Phi \rangle$ and $\langle \chi \rangle$, as per eqs (2.6) and (2.14), are preserved, or at least approximately preserved. Since these VEV patterns are absolutely fundamental to our scheme, the alignment problem is obviously an important one. In this section, we shall briefly discuss this challenge and suggest possible solutions.

First, let us study the most straightforward dynamical completion: a standard renormalisable Higgs potential. The full quartic $G_{SM} \otimes A_4$-invariant Higgs potential in Φ, ϕ

and χ is displayed in appendix B. As expected, it has a large number of terms (more than two dozen) and so may not be compelling as a serious model for nature. Nevertheless, it is amenable to analysis, and it highlights the vacuum alignment challenge.

In the spirit of the "parallel symmetry-breaking worlds" paradigm, it is useful to think of the Higgs potential as the sum of several pieces,

$$V = V(\Phi) + V(\chi) + V(\phi) + V(\Phi, \chi) + V(\Phi, \phi) + V(\phi, \chi) + V(\Phi, \chi, \phi), \qquad (5.1)$$

where the first three terms are the self-interactions of the three Higgs multiplets, while the remaining terms describe interactions between them in an obvious notation.

It is easy to check that eq. (2.6) can be a global minimum of $V(\Phi)$, and that eq. (2.14) can be a global minimum of $V(\chi)$ (and, of course, $V(\phi)$ is manifestly well-behaved since ϕ is a flavour singlet). In all cases, the required Higgs potential parameter space is large.

This is no longer the situation once interactions between Φ and χ are switched on via $V(\Phi, \chi)$. The problem is that the extremum conditions furnish a larger number of independent equations than there are unknown VEVs (v, v_χ and v_ϕ). This means that unnatural fine-tuning conditions have to be enforced on the Higgs potential parameters. The troublesome interaction terms all reside in $V(\Phi, \chi)$, so the most straightforward approach to solving the problem is to find models in which these interactions naturally vanish.[4]

Before suggesting natural ways to make $V(\Phi, \chi) = 0$, we very briefly digress to highlight the importance of the $\Phi - \phi$ interaction term

$$\lambda_3^{\Phi\phi}(\Phi^\dagger\phi) \cdot (\Phi^\dagger\phi) + h.c. \qquad (5.2)$$

If it is nonzero, then the additional global symmetry $U(1)_X$ is explicitly broken to a Z_2 subgroup under which $\phi \to -\phi$, and ℓ_L and all three right-handed charged-lepton fields also change sign. This explicit breaking removes the phenomenologically dangerous potential Goldstone boson, while leaving a discrete subgroup to ensure the absence of the $\overline{\ell}_L \nu_R \Phi$ Yukawa term.

One way to solve the vacuum alignment problem is to introduce additional symmetries to enforce $V(\Phi, \chi) = 0$. The difficulty here is that the most obvious candidate transformations cannot enforce this condition. For the case of a real χ field, the only additional internal transformation allowed is simply $\chi \to -\chi$. But terms of the form $\Phi^\dagger\Phi\chi^2$ are always invariant under this transformation. Similarly, $\Phi \to e^{i\theta}\Phi$ transformations are always invariances. If the model is extended by making χ complex, the natural transformation $\chi \to e^{i\alpha}\chi$ cannot forbid $\Phi^\dagger\Phi\chi^\dagger\chi$ terms.

We have thought of two generic symmetry principles that have potential application to this problem. They both involve spacetime transformations, as purely internal ones do not appear to have sufficient power, as explained above.

The first possibility is to consider the limit where χ completely decouples from the rest of the fields, because (in the absence of gravity) the theory is then invariant under

---

[4]It is worth noting that nonzero values for $\lambda_1^{\Phi\chi}$ and $\lambda_4^{\Phi\chi}$ only are consistent with the extremisation conditions, so having a completely vanishing $V(\Phi, \chi)$ may not be strictly necessary.

independent Lorentz transformations for χ, on the one hand, and the rest of the fields, on the other.[5] The decoupling of χ is achieved in the limit

$$\lambda_\chi \to 0, \qquad \lambda^{\phi\chi} \to 0. \qquad (5.3)$$

We do not, however, wish λ_χ to be precisely zero, otherwise the neutrinos would be exactly degenerate. If λ_χ is small but nonzero, it might be possible to generate an acceptable neutrino mass spectrum while radiatively inducing sufficiently small $V(\Phi,\chi)$ terms, where "sufficiently small" means that those terms alter the required VEV pattern by only a small amount, hence preserving the benefits of that symmetry breaking pattern. A detailed analysis of this possibility is beyond the scope of this paper.

The second spacetime symmetry principle is supersymmetry, acting in concert with internal symmetries. Quartic $V(\Phi,\chi)$ terms can only arise from F-terms. Since the superpotential is at most cubic, this means the generic interaction term is of the form $\Phi_u\Phi_d\chi$, where $\Phi_{u,d}$ are the two Higgs chiral superfields required by supersymmetry, and χ now represents the chiral superfield containing the scalar component of the same name. Internal transformations *can* forbid that cubic superpotential term, thus also forbidding quartic $V(\Phi,\chi)$ terms. (For definiteness, think of $\chi \to -\chi$, though the actual transformations in a realistic supersymmetric extension are almost certainly going to be more involved.) The cubic terms in $V(\Phi,\chi)$ are soft supersymmetry breaking terms such as $\Phi_{u,d}^\dagger\Phi_{u,d}\chi$ and $\Phi_u\Phi_d\chi$, which can also be forbidden by suitable internal (probably discrete) symmetries such as $\chi \to -\chi$. An attempt to construct such a supersymmetric dynamical completion is discussed in appendix C. It serves as an existence proof that a dynamical completion is possible. We have not as yet tried to optimise the model building process, a topic we hope to return to in a future paper.

Another approach worth pursuing, very different from the above, is to take the "parallel worlds of symmetry breaking" language literally, by sequestering [12] Φ and χ on different branes in an extra-dimensional setting (see ref. [2] for an example of a brane-world approach to the neutrino problem). The physical separation of Φ and χ is another generic way to forbid the problematic interaction terms.

The dynamical completion issue should, however, not distract us from the main point of this paper: the A_4 flavour symmetry breaking structure outlined in the preceeding sections.

6. Conclusion

We have proposed an A_4 flavour structure that fits very well with the observed patterns of quark and neutrino mixing, while leaving mass eigenvalues arbitrary. The A_4 field content and Higgs VEV patterns were selected to produce, at lowest order, neutrino tribimaximal

---

[5]This fact may be unfamiliar, though it has been shown to be of relevance for the invisible axion model [11]. It is clear that if all interaction terms between χ and everything else are switched off, then they cannot be generated radiatively. Therefore, in the absence of gravity, there should be an increase in the symmetry of the theory as the χ-decoupling limit is taken. That symmetry increase is for Lorentz transformations of χ to be independent of those for all the other fields.

mixing and zero quark mixing. The required structure splits naturally into two sectors: the neutrino/χ/ϕ domain and the charged-fermion/Φ domain. Different spontaneous flavour breaking patterns occur in these parallel worlds of symmetry breaking. The charged-fermion sector has $A_4 \to C_3$, while the neutrinos see $A_4 \to Z_2$. Radiative or higher-order effects *within* each sector that violate A_4 but preserve the respective subgroup were examined. The C_3 symmetry prevents the generation of quark mixing, while a small and interesting deviation from tribimaximal mixing is allowed by the Z_2. The latter includes a small but nonzero U_{e3} and has CP violation in neutrino oscillations. For quark mixing to be induced, C_3 has to be broken. We explored the natural possibility that the neutrino sector communicates its C_3 breaking to the quarks, and showed through an effective operator analysis that generically this does induce a realistic CKM matrix. Finally, we discussed the dynamical completion challenge and supplied an existence proof that it can be met. Finding the most elegant possible underlying dynamics remains a goal for the future.

We believe that the proposed flavour symmetry structure is a promising base from which to explore the fundamental origin of quark and lepton mixing angles.

Acknowledgments

A. Basic A_4 properties

The alternating group of order four, denoted A_4, is defined as the set of all twelve even permutations of four objects. It has a real three-dimensional irreducible representation $\underline{\mathbf{3}}$, and three inequivalent one-dimensional representations $\underline{\mathbf{1}}$, $\underline{\mathbf{1}}'$ and $\underline{\mathbf{1}}''$. The representation $\underline{\mathbf{1}}$ is trivial, while $\underline{\mathbf{1}}'$ and $\underline{\mathbf{1}}''$ are non-trivial and complex conjugates of each other.

The twelve representation matrices for $\underline{\mathbf{3}}$ are conveniently taken to be the 3×3 identity matrix 1, the reflection matrices $r_1 \equiv \mathrm{diag}(1,-1,-1)$, $r_2 \equiv \mathrm{diag}(-1,1,-1)$ and $r_3 \equiv \mathrm{diag}(-1,-1,1)$, the cyclic and anticyclic matrices

$$c = a^{-1} \equiv \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad a = c^{-1} \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \tag{A.1}$$

respectively, as well as $r_i c r_i$ and $r_i a r_i$. Under the group element corresponding to $c(a)$, $\underline{\mathbf{1}}' \to \omega(\omega^2)\underline{\mathbf{1}}'$ and $\underline{\mathbf{1}}'' \to \omega^2(\omega)\underline{\mathbf{1}}''$, where $\omega = e^{i2\pi/3}$ is a complex cube-root of unity, with both being unchanged under the r_i.

The basic non-trivial tensor products are

$$\underline{\mathbf{3}} \otimes \underline{\mathbf{3}} = \underline{\mathbf{3}}_s \oplus \underline{\mathbf{3}}_a \oplus \underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'', \quad \text{and} \quad \underline{\mathbf{1}}' \otimes \underline{\mathbf{1}}' = \underline{\mathbf{1}}'', \tag{A.2}$$

where $s(a)$ denotes symmetric (antisymmetric) product. Let (x_1, x_2, x_3) and (y_1, y_2, y_3) denote the basis vectors for two $\underline{\mathbf{3}}$'s. Then

$$(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{3}}_s} = (x_2 y_3 + x_3 y_2 \, , \, x_3 y_1 + x_1 y_3 \, , \, x_1 y_2 + x_2 y_1), \tag{A.3}$$

$$(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{3}}_a} = (x_2 y_3 - x_3 y_2 \, , \, x_3 y_1 - x_1 y_3 \, , \, x_1 y_2 - x_2 y_1), \tag{A.4}$$

$$(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{1}}} = x_1 y_1 + x_2 y_2 + x_3 y_3, \tag{A.5}$$

$$(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{1}}'} = x_1 y_1 + \omega \, x_2 y_2 + \omega^2 \, x_3 y_3, \tag{A.6}$$

$$(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{1}}''} = x_1 y_1 + \omega^2 \, x_2 y_2 + \omega \, x_3 y_3, \tag{A.7}$$

in an obvious notation.

B. Higgs potential

The $G_{SM} \otimes A_4$-invariant, renormalisable Higgs potential terms consistent with the discrete Z_2 subgroup of $U(1)_X$ are given by

$$
\begin{aligned}
V(\Phi) &= \mu_\Phi^2 (\Phi^\dagger \Phi)_{\underline{\mathbf{1}}} + \lambda_1^\Phi (\Phi^\dagger \Phi)_{\underline{\mathbf{1}}} (\Phi^\dagger \Phi)_{\underline{\mathbf{1}}} + \lambda_2^\Phi (\Phi^\dagger \Phi)_{\underline{\mathbf{1}}'} (\Phi^\dagger \Phi)_{\underline{\mathbf{1}}''} \\
&\quad + \lambda_3^\Phi (\Phi^\dagger \Phi)_{\underline{\mathbf{3}}_s} (\Phi^\dagger \Phi)_{\underline{\mathbf{3}}_s} + \lambda_4^\Phi (\Phi^\dagger \Phi)_{\underline{\mathbf{3}}_a} (\Phi^\dagger \Phi)_{\underline{\mathbf{3}}_a} \\
&\quad + i \lambda_5^\Phi (\Phi^\dagger \Phi)_{\underline{\mathbf{3}}_s} (\Phi^\dagger \Phi)_{\underline{\mathbf{3}}_a}.
\end{aligned}
\tag{B.1}
$$

$$
\begin{aligned}
V(\chi) &= \mu_\chi^2 (\chi\chi)_{\underline{\mathbf{1}}} + \delta^\chi (\chi\chi\chi)_{\underline{\mathbf{1}}} + \lambda_1^\chi (\chi\chi)_{\underline{\mathbf{1}}} (\chi\chi)_{\underline{\mathbf{1}}} + \lambda_2^\chi (\chi\chi)_{\underline{\mathbf{1}}'} (\chi\chi)_{\underline{\mathbf{1}}''} \\
&\quad + \lambda_3^\chi (\chi\chi)_{\underline{\mathbf{3}}} (\chi\chi)_{\underline{\mathbf{3}}}.
\end{aligned}
\tag{B.2}
$$

$$V(\phi) = \mu_\phi^2 (\phi^\dagger \phi) + \lambda^\phi (\phi^\dagger \phi)^2 \tag{B.3}$$

$$
\begin{aligned}
V(\Phi, \chi) &= \delta_s^{\Phi\chi} (\Phi^\dagger \Phi)_{\underline{\mathbf{3}}_s} \chi + i \delta_a^{\Phi\chi} (\Phi^\dagger \Phi)_{\underline{\mathbf{3}}_a} \chi + \lambda_1^{\Phi\chi} (\Phi^\dagger \Phi)_{\underline{\mathbf{1}}} (\chi\chi)_{\underline{\mathbf{1}}} \\
&\quad + \lambda_2^{\Phi\chi} (\Phi^\dagger \Phi)_{\underline{\mathbf{1}}'} (\chi\chi)_{\underline{\mathbf{1}}''} + \lambda_2^{\Phi\chi*} (\Phi^\dagger \Phi)_{\underline{\mathbf{1}}''} (\chi\chi)_{\underline{\mathbf{1}}'} \\
&\quad + \lambda_3^{\Phi\chi} (\Phi^\dagger \Phi)_{\underline{\mathbf{3}}_s} (\chi\chi)_{\underline{\mathbf{3}}} + i \lambda_4^{\Phi\chi} (\Phi^\dagger \Phi)_{\underline{\mathbf{3}}_a} (\chi\chi)_{\underline{\mathbf{3}}}.
\end{aligned}
\tag{B.4}
$$

$$
\begin{aligned}
V(\Phi, \phi) &= \lambda_1^{\Phi\phi} (\Phi^\dagger \Phi)_{\underline{\mathbf{1}}} (\phi^\dagger \phi) + \lambda_2^{\Phi\phi} (\Phi^\dagger \phi)(\phi^\dagger \Phi) + \lambda_3^{\Phi\phi} (\Phi^\dagger \phi)(\Phi^\dagger \phi) \\
&\quad + \lambda_3^{\Phi\phi*} (\phi^\dagger \Phi)(\phi^\dagger \Phi).
\end{aligned}
\tag{B.5}
$$

$$V(\phi, \chi) = \lambda^{\phi\chi} (\phi^\dagger \phi)(\chi\chi)_{\underline{\mathbf{1}}}. \tag{B.6}$$

There is no renormalizable term simultaneously involving Φ, ϕ and χ allowed by the Z_2 subgroup of $U(1)_X$, that is, $V(\Phi, \chi, \phi) = 0$.

The total potential is given by

$$V = V(\Phi) + V(\chi) + V(\phi) + V(\Phi, \chi) + V(\Phi, \phi) + V(\phi, \chi) + V(\Phi, \chi, \phi). \tag{B.7}$$

C. A supersymmetric dynamical completion

For the sake of supplying an existence theorem, we have constructed one example of such a theory. It is rather elaborate, in that it requires an additional discrete $Z_{12} \otimes Z_2$ symmetry

together with supersymmetry and several additional fields. Since χ is now in a supermultiplet it becomes a complex field, and for the usual reason the number of Higgs doublets must be doubled. The chiral superfield content is

$$
\begin{aligned}
&Q_L \sim (\underline{\mathbf{3}}, 1, -1), \qquad u_R^c,\ d_R^c \sim (\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'', \omega_{12}^7, -1), \\
&\ell_L \sim (\underline{\mathbf{3}}, 1, -1), \qquad e_R^c \sim (\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'', \omega_{12}^7, -1), \qquad \nu_R \sim (\underline{\mathbf{3}}, \omega_{12}^4, -1), \\
&\Phi_{u,d} \sim (\underline{\mathbf{3}}, \omega_{12}^5, 1), \qquad \phi_u \sim (\underline{\mathbf{1}}, \omega_{12}^8, 1), \qquad \phi_d \sim (1, \omega_{12}^4, 1) \\
&\chi \sim (\underline{\mathbf{3}}, \omega_{12}^4, 1), \qquad \chi' \sim (\underline{\mathbf{3}}, \omega_{12}^2, 1), \\
&s \sim (\underline{\mathbf{1}}, \omega_{12}^4, 1), \qquad s' \sim (\underline{\mathbf{1}}, \omega_{12}^2, 1), \qquad s'' \sim (\underline{\mathbf{1}}, \omega_{12}^8, 1),
\end{aligned}
\tag{C.1}
$$

where $\omega_{12}^{12} = 1$. The first entry provides the A_4 assignment, the second specifies how that field multiplicatively transforms under the Z_{12}, and the third the parity under the Z_2. The gauge assignments for the quark, lepton and Higgs-doublet superfields are standard, with s, s' and s'' being new gauge and A_4 singlets, while χ' is a new gauge-singlet A_4 triplet.

The Z_{12} charges are chosen in such a way that, first, communication between χ and $\Phi_{u,d}$ is forbidden, so as to avoid the troublesome terms in $V(\Phi, \chi)$, but, second, the χ^3 and $\Phi_u \Phi_d \chi'$ terms are allowed so that the desired VEV alignment can be enforced. The Z_2 charge disallows terms of the type $\nu_R^c \chi^2$ which can cause the fermion partner of χ to mix with neutrinos and therefore destroy the pattern of the neutrino mass matrix.

The superpotential contains the terms $Q_L \Phi_u u_R^c$, $Q_L \Phi_d d_R^c$, $\ell_L \phi_u \nu_R^c$, $\ell_L \Phi_d e_R^c$, $\nu_R^c \nu_R^c s$ and $\nu_R^c \nu_R^c \chi$ which supply all the central Yukawa couplings. There are no bare Majorana masses for ν_R, but $\nu_R^c \nu_R^c s$ generates universal Majorana masses once the spin-0 component of s acquires a VEV.

The superpotential for the Higgs multiplets is

$$
\begin{aligned}
W ={}& a_1 \chi^3 + a_2 \chi^2 s + a_3 s^3 + a_4 \phi_u \phi_d + a_5 (\Phi_u \Phi_d)_{3s} \chi' + a_6 (\Phi_u \Phi_d)_{3a} \chi' \\
&+ a_7 \Phi_u \Phi_d s' + a_8 \chi'^2 s'' + a_9 s s'' + a_{10} s'^2 s'' + a_{11} s''^3.
\end{aligned}
\tag{C.2}
$$

From this structure, it is evident that all supersymmetric $V(\Phi, \chi)$ terms are absent from the F-term contributions, while the D-term contributions are also safe because, of course, they cannot involve the gauge singlet χ.

In the supersymmetric limit, the desired VEV structure cannot be obtained, but supersymmetry has to be broken in any case. To this end, we follow the usual soft supersymmetry breaking approach by adding to the potential all soft-breaking terms that preserve $A_4 \otimes Z_{12} \otimes Z_2$. These terms are given by

$$
\begin{aligned}
V_{soft} ={}& b_1 \chi^3 + b_2 \chi^2 s + b_3 s^3 + b_4 \phi_u \phi_d + b_5 (\Phi_u \Phi_d)_{3s} \chi' + b_6 (\Phi_u \Phi_d)_{3a} \chi' \\
&+ b_7 \Phi_u \Phi_d s' + b_8 \chi'^2 s'' + b_9 s s'' + b_{10} s'^2 s'' + b_{11} s''^3 + H.C. \\
&+ c_1 \chi^\dagger \chi + c_2 s^\dagger s + c_3 s'^\dagger s' + c_4 s''^\dagger s'' + c_5 \phi_u^\dagger \phi_u + c_6 \phi_d^\dagger \phi_d + c_7 \chi'^\dagger \chi' \\
&+ c_8 \Phi_u^\dagger \Phi_u + c_9 \Phi_d^\dagger \Phi_d.
\end{aligned}
\tag{C.3}
$$

We have checked that the total Higgs potential resulting from above admits the required forms for the VEVs as extrema for the case where all the Higgs potential parameters are real. Terms from $a_{1,2}$, $b_{1,2}$ and c_1 allow two solutions for the VEV pattern of χ, namely, that all component VEVs are equal or that only one of the them is nonzero. The latter is the desired one. Terms from $a_{5,6,7,8}$, $b_{5,6,7,8}$ and $c_{7,8,9}$ force the component VEVs to be equal, if nonzero, in each of the fields χ', Φ_u and Φ_d.

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